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سكشن 9 رياضة هندسية

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9

IR.S.P $\quad x\,(P(x))\big|_{x=0} = \infty$

$$x^2\,(Q(x))\big|_{x=0} = \infty$$

ليس لها حل.

R.SP $\quad x\,P(x)\big|_{x=0} \neq \infty$

$$x^2\,(Q(x))\big|_{x=0} \neq \infty$$

$$\boxed{y = \sum a_n x^{n+\lambda}} \rightarrow y_{G.S} = c_1 y_1 + c_2 y_2$$

$$a\lambda^2 + b\lambda + c = 0$$

$\lambda_1 - \lambda_2 \neq$ عدد صحيح

$$y_1 = x^{\lambda_1}\left[a_0 + a_1 x + \dots\right]$$

$$y_2 = x^{\lambda_2}\left[a_0 + a_1 x + \dots\right]$$

$\lambda_1 = \lambda_2 = \lambda$

$$y_1 = x^{\lambda}\left[a_0 + a_1 x + \dots\right]$$

$$y_2 = \frac{\partial y(x,y)}{\partial \lambda}\bigg|_{\lambda}$$

$\lambda_1 - \lambda_2 =$ عدد صحيح

$$y_1 = x^{\lambda_1}\left[a_0 + a_1 x + \dots\right]$$

$a_n \neq \infty$

نكتفي بحل واحد

$a_n = \infty$

$$y_2 = \frac{\partial(\lambda-\lambda_2)\,y}{\partial \lambda}\bigg|_{\lambda=\lambda_2}$$

b

Ex:-

$$x^2 y'' + x y' + \left(x^2 - \frac{4}{9}\right) y = 0$$

$$y'' + \frac{1}{x} y' + \frac{\left(x^2 - \frac{4}{9}\right)}{x^2} y = 0$$

$$\lim_{x \to 0} P(x) = \frac{1}{x} \longrightarrow = \infty$$

$$\lim_{x \to 0} Q(x) = \frac{x^2 - \frac{4}{9}}{x^2} = \infty$$

نضرب $P(x)$ في x $\longrightarrow x \cdot \frac{1}{x} = 1$

نضرب $Q(x)$ في x^2 $\longrightarrow x^2 \cdot \frac{x^2 - 4/9}{x^2} = -4/9$ } R.S.P

$$\therefore\ y = \sum a_n x^{n+\lambda}$$

$$y' = \sum_{n=0}^{\infty} (n+\lambda) a_n x^{n+\lambda-1}$$

$$y'' = \sum_{n=0}^{\infty} (n+\lambda)(n+\lambda-1) a_n x^{n+\lambda-2}$$

$$\sum (n+\lambda)(n+\lambda-1) a_n x^{n+\lambda} + \sum (n+\lambda) a_x x^{n+\lambda} + \sum a_n x^{n+\lambda+2} - \frac{4}{9} \sum a_n x^{n+\lambda} = 0$$

لمعرفة أقل عدد نعوض عن $n=0$

هنشتغل على أول عدد

$$\text{Cof } x^{\lambda} \longrightarrow \lambda(\lambda-1) a_0 + \lambda a_0 - \frac{4}{9} a_0 = 0$$

$$a_0 \left(\lambda^2 \cancel{-\lambda} \cancel{+\lambda} - 4/9\right) = 0$$

إما $a_0 = 0$ وهذا مرفوض

إما $\lambda^2 - 4/9 = 0 \longrightarrow \lambda = \pm 2/3$

الحالة الأولى ∴ $\Longleftarrow$ $\lambda_1 - \lambda_2 =$ عدد غير صحيح

$$\text{Co.of } x^{\lambda+1} \longrightarrow (\lambda+1)(\lambda)a_1 + (\lambda+1)a_1 - \tfrac{4}{9}a_1 = 0$$

$$a_1(\lambda^2 + \lambda + \lambda + 1 - \tfrac{4}{9}) = 0$$

إما

$\boxed{a_1 = 0}$

(✓)

إما ده $= 0$

(×)

$$\text{Co.of } x^{n+\lambda} \longrightarrow (n+\lambda)(n+\lambda-1)a_n + (n+\lambda)a_n + a_{n-2} - \tfrac{4}{9}a_n = 0$$

$$a_n = \frac{-(a_{n-2})}{(n+\lambda)(n+\lambda-1) + (n+\lambda) - 4/9} = \frac{-(a_{n-2})}{(n+\lambda)^2 - 4/9}$$

$$\text{at } n=2 \qquad a_2 = \frac{-a_0}{(\lambda+2)^2 - 4/9}$$

$$\therefore\ y_1 = x^{\frac{2}{3}}\left[a_0 - \frac{a_0}{(\frac{2}{3}+2)^2 - 4/9}x^2 + \cdots\cdots\right]$$

$$y_2 = x^{-\frac{2}{3}}\left[a_0 - \frac{a_0}{(\frac{-2}{3}+2)^2 - 4/9}x^2 + \cdots\cdots\right]$$

$$y_{G.S} = c_1 y_1 + c_2 y_2$$

d

$$x^2 y'' - x y' + (1+x) y = 0$$

$$P(x) = \frac{-1}{x} = \infty \quad * \, x = -1 \neq \infty$$

$$Q(x) = \frac{1+x}{x^2} = \infty \quad * \, x^2 = 1 \neq \infty$$

R.S.P

$$y = \sum a_n x^{n+\lambda}$$

$$y' = \sum (n+\lambda) a_n x^{n+\lambda-1}$$

$$y'' = \sum (n+\lambda)(n+\lambda-1) a_n x^{n+\lambda-2}$$

$$\sum (n+\lambda)(n+\lambda-1) a_n x^{n+\lambda} - \sum (n+\lambda) a_n x^{n+\lambda} + \sum a_n x^{n+\lambda} + a_n x^{n+\lambda+1} = 0$$

نعوض n = 0 $\longrightarrow$ $x^{\lambda} \rightarrow$ أقل أس

$$\text{Co. of } x^{\lambda} \longrightarrow \lambda * (\lambda-1) a_0 - \lambda a_0 + a_0 = 0$$

$$a_0 \underbrace{(\lambda^2 - \lambda - \lambda + 1)}_{=0} = 0$$

$$\lambda^2 - 2\lambda + 1 = 0$$

$$\lambda_1 = \lambda_2 = \lambda = 1 \longrightarrow$$ في الحالة الثانية

$$\text{Co. of } x^{\lambda+1} \longrightarrow (\lambda+1) * \lambda a_1 - (\lambda+1) a_1 + a_1 + a_0 = 0$$

$$a_1 = \frac{a_0}{(\lambda^2 + \cancel{\lambda} - \cancel{\lambda} - \cancel{1} + 1)} = \frac{a_0}{\lambda^2}$$

Co. of $x^{n+\lambda}$

$$\longrightarrow (n+\lambda)(n+\lambda-1)a_n - (n+\lambda)a_n + a_n + a_{n-1} = 0$$

$$a_n = \frac{-(a_{n-1})}{(n+\lambda)(n+\lambda-1) - (n+\lambda) + 1}$$

$$a_n = \frac{-(a_{n-1})}{(n+\lambda)(n+\lambda-1-1)+1} = \frac{-(a_{n-1})}{(n+\lambda)(n+\lambda-2)+1}$$

$$a_2 = \frac{-a_1}{(\lambda+2)(\lambda)+1}$$

$$y_1 = x\left[a_0 - \frac{a_0}{1}x - \frac{a_1}{4}x^2 + \cdots\right]$$

$$y(x,\lambda) = x^{\lambda}\left[a_0 - \frac{a_0}{\lambda^2}x + \frac{a_0}{\lambda^3(\lambda+2)+\lambda^2}x^2 + \cdots\right]$$

$$y_2 = \frac{\partial y(x,\lambda)}{\partial\lambda} = x^{\lambda}\left[+2a_0 x\lambda^{-3} + \cdots\cdots\right] +$$

$$x^{\lambda} * 1 * \ln\lambda \left[a_0 - \frac{a_0}{\lambda}x - \frac{a_1}{4}x^2 + \cdots\right]$$

نفسها   تفاضلها اسها   أسها

F **Ex₂** $x^2 y'' + (x^2 - 2x) y' + 2y = 0$

$$P(x) = \frac{x^2 - 2x}{x^2} = \infty$$

$$Q(x) = \frac{2}{x^2} = \infty$$

$$* x \longrightarrow \frac{x^2 - 2x}{x} = -2 \neq \infty$$

$$* x^2 \longrightarrow 2 = \infty$$

R.S.P

$$y = \sum a_n x^{n+\lambda}$$

$$y' = (n+\lambda) a_n x^{n+\lambda-1}$$

$$y'' = (n+\lambda)(n+\lambda-1) a_n x^{n+\lambda-2}$$

$$\sum (n+\lambda)(n+\lambda-1) a_n x^{n+\lambda} + \sum (n+\lambda) a_n x^{n+\lambda+1}$$

$$-2 \sum (n+\lambda) a_n x^{n+\lambda} + 2 \sum a_n x^{n+\lambda} = 0$$

co-of $x^{\lambda} \longrightarrow \lambda(\lambda-1) a_0 - 2\lambda a_0 + 2 a_0 = 0$

$$a_0 (\lambda^2 - \lambda - 2\lambda + 2) = 0$$

$\lambda = 1, 2$ = عدد صحيح ← في الحالة الثالثة

co-of $x^{\lambda+1} \longrightarrow (\lambda+1)(\lambda) a_1 + \lambda a_0 - 2(\lambda+1) a_1$

$+ 2 a_1 = 0$

$$a_1 = \frac{-\lambda a_0}{(\lambda^2 + \lambda - 2\lambda - 2 + 2)} = \frac{-a_0}{\lambda + 1}$$

Co-oF $x^{n+\lambda}$

$$(n+\lambda)(n+\lambda-1)a_n + (n+\lambda-1)a_{n-1} - 2(n+\lambda)a_n + 2a_n = 0$$

$$a_n = \frac{-(n+\lambda-1)a_{n-1}}{(n+\lambda)(n+\lambda-1) - 2(n+\lambda) + 2} = \frac{-(n+\lambda-1)a_{n-1}}{(n+\lambda)(n+\lambda-3)+2}$$

$$a_2 = \frac{-(\lambda+1)a_1}{(\lambda+2)(\lambda-1)+2}$$

نكملها في الصفحة التالية